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Enhancement of the Kalman Filter Performance in Guidance Application

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Abstract—Kalman filter algorithm is an estimator known as the workhorse of estimation. It has an important application in missile guidance, especially in lack of accurate data of the target due to noise or uncertainty. In this paper a Kalman filter is used as a tracking filter in a simulated target-interceptor scenario with noise. It estimates the position, velocity, and acceleration of the target in the presence of noise. These estimations are needed for both proportional navigation and differential geometry guidance laws. A Kalman filter has a good performance at low noise, but a large noise causes considerable errors leads to performance degradation. Therefore, a new technique is required to overcome this defect using tuning factors to tune a Kalman filter to adapt increasing of noise. The values of the tuning factors are between 0.8 and 1.2, they have a specific value for the first half of range and a different value for the second half. they are multiplied by the estimated values. These factors have its optimum values and are altered with the change of the target heading. A genetic algorithm updates these selections to increase the maximum effective range which was previously reduced by noise. The results show that the selected factors have other benefits such as decreasing the minimum effective range that was increased earlier due to noise. In addition to, the selected factors decrease the miss distance for all ranges of this direction of the target, and expand the effective range which leads to increase probability of kill.

Index Terms — Kalman filter, Genetic algorithm, guidance law, Proportional navigation, Differential geometry, 3DOF

1 Introduction

KALMAN filter is a state estimator applied to a process that has random states. It estimates minimum error variance of the unknown state of a dynamic system taken from noisy data at discrete real-time. It has been widely used in many areas of industrial applications such as video and laser tracking systems, ballistic missile trajectory estimation, radar, and fire control, also the Kalman filter has become more useful even for the complicated applications because of the developments of high-speed computers [1].

One of the most important applications of state estimation is moving object tracking known as the process of determine the future states of moving object which can be done using Kalman filter. In last decade there have been a lot or researches on the tracking of moving objects within a scene. Systems modified for such tasks as people tracking [2] facial tracking [3] and moving vehicles tracking [4], [5] and [6] have come in many shapes and size. In [7] the Kalman Filter with optimal control approach was used to control the angle of attack of a missile.

In this paper, a Kalman filter is used as a tracking filter in three degree of freedom (3DOF) target-interceptor scenario. It estimates the data concerning target path which required by guidance law to control the interceptor trajectory into the direction of intercept point. Estimated values will multiply by tuning factor to enhance the performance in the presence of noise and changing these factors in case of changing noise gain or target heading. The tuned factors are selected and optimized using genetic algorithm search technique to reach the optimum performance. The (3DOF) model, Kalman filter, binary genetic algorithm are simulated using Matlab program.

The rest of the paper is organized as follows, in second section a three degree of freedom (3DOF) model is illustrated. In section three the guidance laws are briefly described while in section four and Kalman filtering steps are outlined. Section five explains a genetic algorithm steps to select the optimum values of tuning factors used by both proportional navigation and differential geometry guidance laws. Conclusions are outlined on section seven. The last section is the summary.

2 Three degree of freedom (3DOF) model

Fig.1 shows the block diagram of the target-missile model which has the following parts

2.1 Missile motion

The interceptor is modeled as a point in NED (North-East-Down) coordinate system with the assumption that the earth is flat. The following vector describes the missile present state

$$M(k) = [x_M(k) \quad \dot{x}_M(k) \quad y_M(k) \quad \dot{y}_M(k) \quad z_M(k) \quad \dot{z}_M(k)]^T \quad (1)$$

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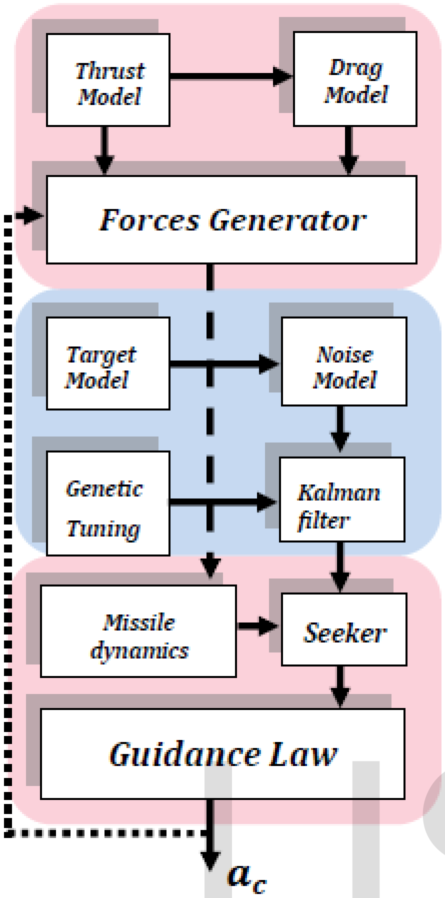

Fig.1 Block diagram of target-interceptor scenario

The next state is described by

$$M(k+1) =$$

$$\begin{bmatrix} 1 & \Delta & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_M(k) \\ \dot{x}_M(k) \\ y_M(k) \\ \dot{y}_M(k) \\ z_M(k) \\ \dot{z}_M(k) \end{bmatrix} + \begin{bmatrix} 0.5\ddot{x}_M(k)\Delta^2 \\ \ddot{x}_M(k)\Delta \\ 0.5\ddot{y}_M(k)\Delta^2 \\ \ddot{y}_M(k)\Delta \\ 0.5\ddot{z}_M(k)\Delta^2 \\ \ddot{z}_M(k)\Delta \end{bmatrix} \quad (2)$$

Where Δ is discrete time step size

2.2 Target motion

The present state of the target is represented by

$$T(k) = [x_T(k) \quad \dot{x}_T(k) \quad y_T(k) \quad \dot{y}_T(k) \quad z_T(k) \quad \dot{z}_T(k)]^T \quad (3)$$

The next state is

$$T(k+1) = \begin{bmatrix} 1 & \Delta & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_T(k) \\ \dot{x}_T(k) \\ y_T(k) \\ \dot{y}_T(k) \\ z_T(k) \\ \dot{z}_T(k) \end{bmatrix} \quad (4)$$

In three second before the impact, the target begins a six g turn in the x-y plane. In this case the modified next state is

$$T(k+1) = \begin{bmatrix} 1 & A & 0 & B & 0 & 0 \\ 0 & C & 0 & -D & 0 & 0 \\ 0 & B & 1 & A & 0 & 0 \\ 0 & D & 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_T(k) \\ \dot{x}_T(k) \\ y_T(k) \\ \dot{y}_T(k) \\ z_T(k) \\ \dot{z}_T(k) \end{bmatrix} \quad (5)$$

$$A = \frac{\sin(\omega_t\Delta)}{\omega_t}, \; B = \frac{\{1-\cos(\omega_t\Delta)\}}{\omega_t}, \; C = \cos(\omega_t\Delta),$$

$$D = \sin(\omega_t\Delta), \quad \omega_t = \frac{a_{turn}}{\sqrt{\dot{x}_T(k)^2+\dot{y}_T(k)^2}}$$

where ω_t is the target angular turn rate, $a_{turn} = 6g$

2.3Missile thrust and drag

1- The thrust is 23000 Newton for the first 6 seconds. It accelerates the missile up to 1100 m/sec.
2- The drag is composed of induced drag and parasitic drag. The induced drag due to shape of missile and dynamic pressure is

$$F_{di} = Q * 0.25 \frac{m\sqrt{a_{Ma}^2+(a_{Me}-g)^2}}{m\,g} S_{ref} \quad (6)$$

Where m is the mass of the missile, a_{Ma} and a_{Me} are azimuth and elevation command accelerations for the missile, g is the gravitational acceleration, S_{ref} is the cross sectional area of the missile, Q is a dynamic pressure. The parasitic drag is

$$F_{dp} = Q * C_{dp} * S_{ref} \quad (7)$$

C_{dp} is a parasitic drag coefficient. The total drag is the sum of induced drag and parasitic drag.

3 Guidance Laws

3.1 Proportional navigation (PN) law

Fig. 2 shows the engagement geometry for PN law. The main concept of proportional navigation is to keep line of sight constant by eliminating line of sight rate. It can be expressed as

$$a_c(t) = N v_{cl} \dot{\lambda} \quad (8)$$

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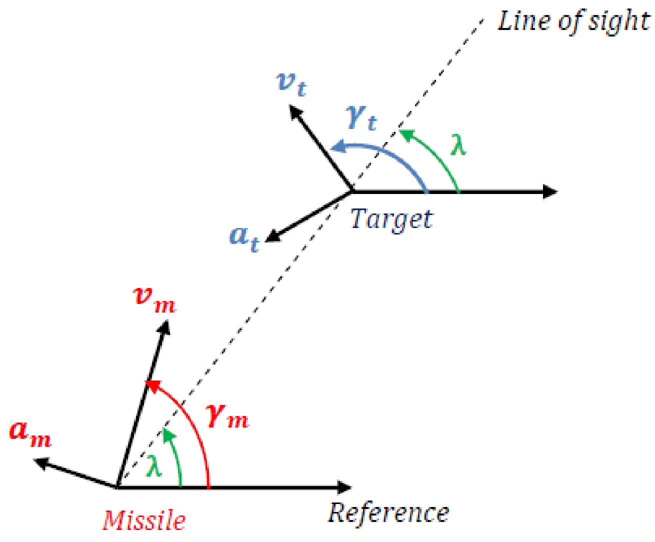

Fig.2 Engagement geometry of PN law

,where $a_c(t)$ is the acceleration command, N is the navigation ratio, v_{cl} is the estimated closing velocity, $\dot{\lambda}$ is the estimated line of sight rate. In the case of Kalman filter tuning, this guidance law can modified to the form

$$a_c(t) = N(T_{f1} v_{cl})(T_{f2} \dot{\lambda}) \tag{9}$$

Where T_{f1} and T_{f2} are the tuning factors for both estimated closing velocity and estimated line of sight rate respectively, and there values lies between (0.8,1.2) and genetic algorithm is used to find the optimum values of them.

3.2 Differential geometry (DG) law

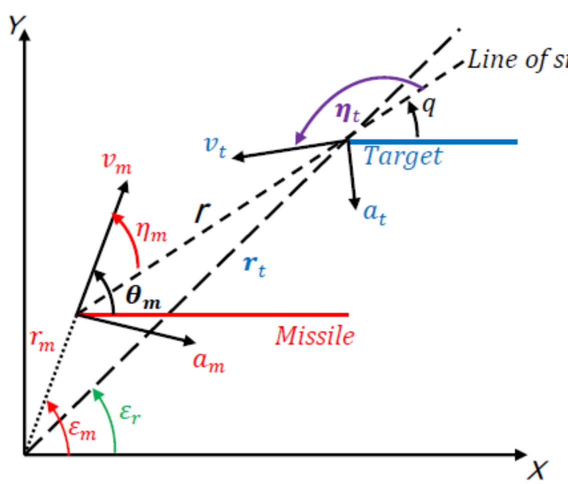

Fig.3 Engagement geometry of DG law

Fig.3 shows the engagement for DG law. It can be expressed as

$$a_c(t) = \|a_t\| \frac{\cos \eta_t}{\cos \eta_m} + N \frac{v_{cl} \dot{\lambda}}{\cos \eta_m} \tag{10}$$

where a_t is a target acceleration, η_t is a target heading relative to line of sight, η_m is a missile look angle, N is a navigation constant. In the case of Kalman filter tuning, this guidance law can modified to be

$$a_c(t) = \|T_{f1} a_t\| \frac{\cos(T_{f2} \eta_t)}{\cos \eta_m} + N \frac{(T_{f3} v_{cl})(\dot{\lambda} T_{f4})}{\cos \eta_m} \tag{11}$$

4 Kalman Filter

The Kalman filtering algorithm is a state estimator known as the workhorse of estimation. In this paper, it is used to estimate the position, velocity, and acceleration of noisy estimation of position. It can be broken into prediction and correction phases [8].

At the end of the current state k the prediction phase occurs. It computes a next state (k+1) parameters and covariance using the current (k) state estimate. The measurements are taken between the prediction and correction phases, and then the correction phase calculates a corrected state estimate and covariance for time (k +1).

4.1 Noise modeling

Noisy line of sight angle and range are generated with

$$\theta_{Lnz} = \theta_L + \sigma_{\theta_L} n_{rand} \tag{12}$$

$$r_{nz} = r + \sigma_r n_{rand} \tag{13}$$

n_{rand} is a random value, drawn from a zero mean Gaussian distribution with variance of one. The standard deviations are

$$\sigma_{\theta_L} = f_{noise} \sigma_{\theta_L\_base} \tag{14}$$

$$\sigma_r = f_{noise} \sigma_{r\_base} \tag{15}$$

f_{noise} is a factor used to determine the gain of noise. The baseline range and line of sight angle standard deviations are defined as 10 meters and 1 m.rad respectively, [9].

The guidance system of the missile responds to noise with slight guidance accelerations commands which produce an induced drag. Thus the total drag increases exponentially with increasing of missile velocity which leads to produce dramatic performance deterioration.

4.2 Predication phase

The state prediction equation is

$$\hat{x}(k+1|k) = F(k)\hat{x}(k|k) + G(k)u(k) \tag{16}$$

$F(k)$ is the state transition matrix, u(k) is a deterministic input vector, and G(k) is an identity matrix used to weight input.

The predicted state estimate covariance estimates the accuracy of the predicted state estimate based on the corrected state covariance and process covariance of the previous time step k. It is given by

$$P(k+1|k) = F(k)P(k|k)F(k)' + Q(k). \quad (17)$$

Q(k) is a process covariance. The predicted measurement estimate is defined as the filter expectation of the state (k+1) measurement based on the predicted state estimate, It is given by

$$\hat{z}(k+1|k) = H(k+1)\hat{x}(k+1|k) \quad (18)$$

$H(k+1)$ is the measurement extraction matrix which represents the measurement from the state vector for time (k+1).

4.3 Correction phase

In this phase the corrected state estimate and covariance are generated by updating the predicted state estimate and covariance. This phase contains calculation of residual of the measurements which is defined as the difference between the real measurements and the predicted measurements

$$r(k+1) = z(k+1) - \hat{z}(k+1|k) \quad (19)$$

The measurement $z(k+1)$represents state translation and transients during the time elapsed between steps as well as measurement noise. It can be represented as

$$z(k+1) = H(k+1)x(k+1) + w(k+1) \quad (20)$$

$w(k+1)$ is the filter gain. It determines the influence of previous estimates and the current measurement on the predicted state estimate and covariance.

$$W(k+1) = P(k+1|k)H(k+1)'S(k+1)^{-1} \quad (21)$$

The covariance of the measurement residual is

$$S(k+1) = R(k+1) + H(k+1)P(k+1|k)H(k+1)' \quad (22)$$

The next step is correction of state estimation. It modifies the predicted state by a magnitude according to the measurement residual and the filter gain, it is

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + W(k+1)r(k+1) \quad (23)$$

Finally, the corrected state estimation covariance is

$$P(k+1|k+1) = P(k+1|k) - W(k+1)S(k+1)W'(k+1) \quad (24)$$

5 Optimization Process

Fig.4 shows the flow chart of the optimization process.
First, in the case of proportional navigation there are two factors to tune the estimated values of line of sight rate and closing velocity. The tuning factors have specific constant values for the half of range and new constant values for the rest of the range until interception point. So the optimization problem in this case is to find the optimum values for four variables which achieve the desired cost function.
Second, in the case of differential geometry (DG) guidance law, the scale of the optimization problem is two time the preceding one. Since the estimated values to be tuned are target acceleration, target lead angle, closing velocity, and line of sight rate. Every estimated value has its tuning factor that is divided into two values as explained in PN law. Thus the optimization problem in DG law is to find the optimum values for eight variables that achieve the desired cost function.
The objective function is to decrease the miss distance at long ranges to achieve miss distance less than kill radius (5m) which also decrease the miss distance at less ranges at the same target direction and leads to reduce the minimum effective range below the minimum value. An optimization process is formulating using binary genetic algorithm Matlab program in reference [10].

6 Simulation Results

6.1 KF performance with proportional navigation law

6.1.1 The effect of noise on kinematic boundaries

The main object of Kalman filter is estimating the target range and line of sight. The estimation errors are increased in the presence of noise and causes performance degradation like decreasing the maximum effective range, increasing minimum effective range and increasing the miss distance.
Fig.5 shows the effect of noise of the maximum effective range with different values of noise factors. The values of maximum effective range are taken every thirty degrees and beginning with zero up to 180 degree and drawing a mirror figure from 180 to 360 degree. The figure shows performance degradation in the direction of 30 degree and 60 degrees due to estimation errors.

6.1.2 Tuning of KF performance for noise factor=2

To enhance the performance of PN law, the estimates values for both ranges and line of sight are multiplied by correction factors to eliminate estimation errors. The correction factor of range is divided into two values for the first half and the second half of range. The correction factor of line of sight is divided with the same way. The values of the correction factor lie between 0.8 and 1.2. The genetic algorithm is used to find the optimum values of the correction factors.

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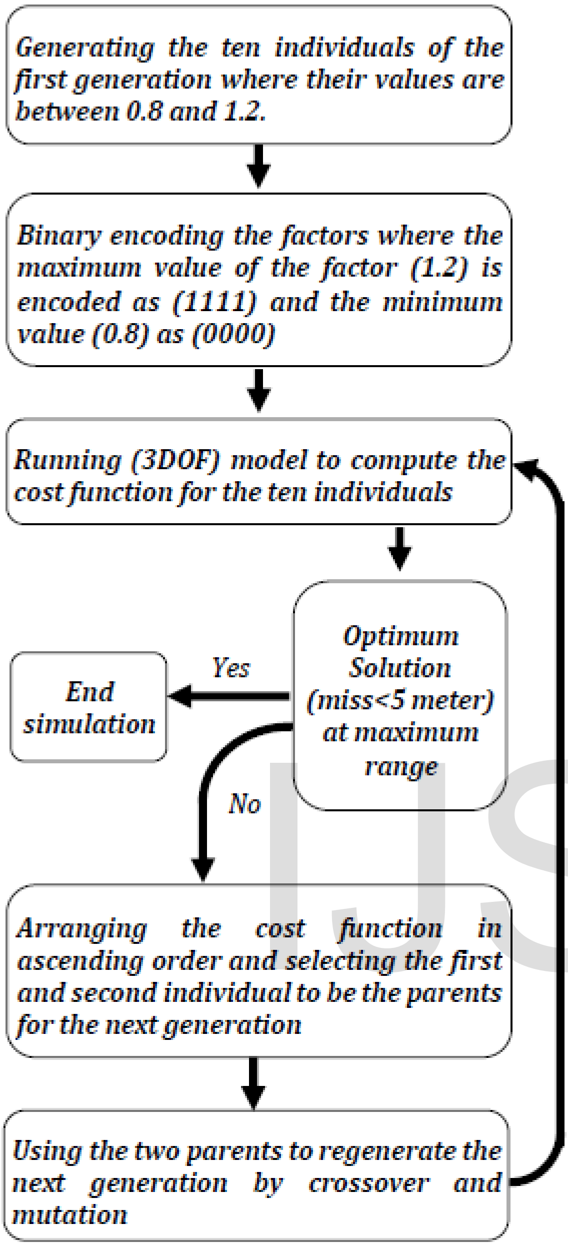

Fig.4 Flow chart of optimization process

Fig.6 shows the performance of Kalman filter and tuned Kalman filter in the direction of 30 degree and noise factor=2 for low ranges. The figure shows that no change in the minimum effective range which is equal to five kilometers but the average values of miss distances which resulting from run the simulation 20 times are decreased at the ranges (1, 2, and 3 kilometers).

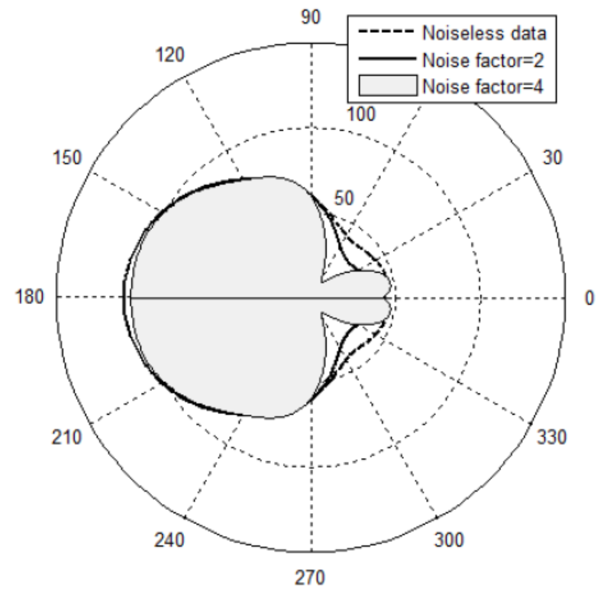

Fig.5 PN performance at different values of noise factor

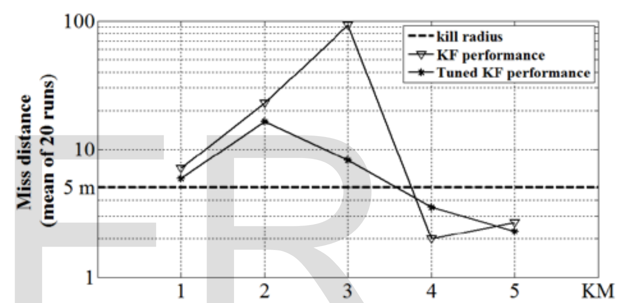

Fig.6 KF performance for at low ranges (NF=2)

Fig.7 has the same comparison as Fig.6 but at long ranges. The figure shows that tuning Kalman filter performance increases the maximum effective range from 40 Km to 41 Km and reduces the miss distance at a lot of ranges.

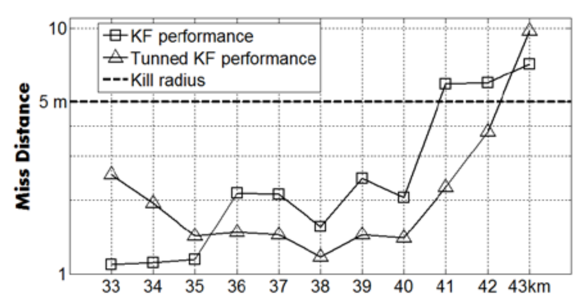

Fig.7 KF performance at long ranges (NF=2)

Fig.8 contains the values of multiplied factors for both LOS estimated and range estimated and their distribution along the range between target and interceptor

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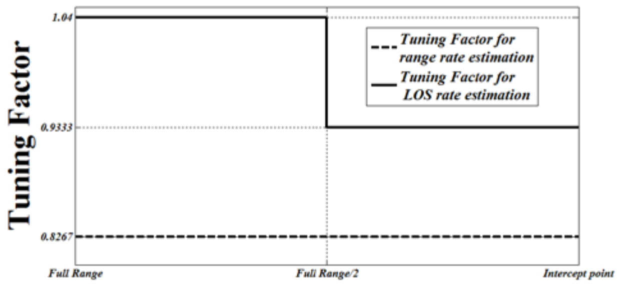

Fig.8 Tuning factors distribution along the range (NF=2)

6.1.3 Tuning of KF performance for noise factor=4

The noise affects the performance of Kalman filter, so increasing noise factor causes performance degradation of Kalman filter and it needs to be tuned again to adapt increasing the noise gain. Therefore the same steps are taken again using genetic algorithm to find the optimum values of tuning factor

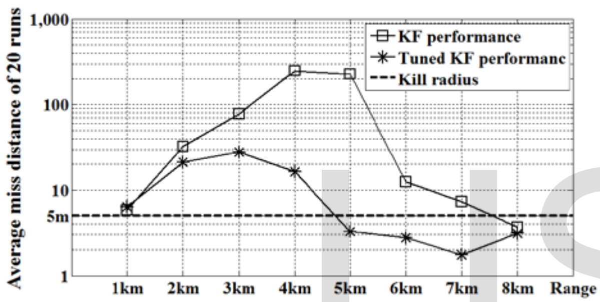

Fig.9 KF performance at low ranges (NF=4)

Fig.9 indicates the values of the average miss distances of 20 runs at low ranges. The figure shows that tuning Kalman filter reduce the minimum effective range from 8 to 5 Km and also minimizes the miss distances at all ranges for noise factor=4. Fig.10 KF performance at long ranges (NF=4)

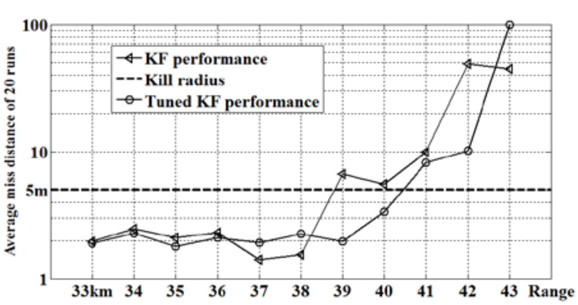

Fig.10 KF performance at long ranges (NF=4)

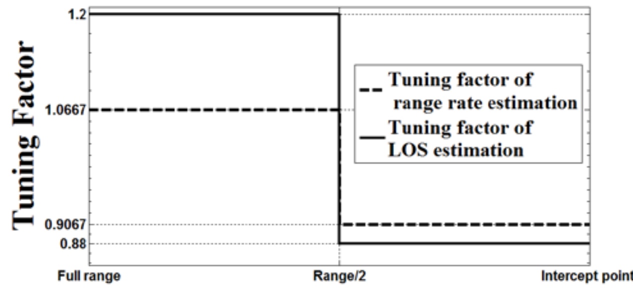

Fig.11 Tuning factor distribution along the range (NF=4)

Fig.10 shows the same data but for long ranges which also indicates enhancement of maximum effective range from 38 to 40 Km. Fig.11 shows the tuning factors that achieve the optimum performance for both low and long ranges.

Fig.12 shows 3D representation of genetic algorithm search results of range 40 Km, heading 30 degree, and noise factor=4. The optimum (minimum) average miss distance is the first individual of the seventeenth generation

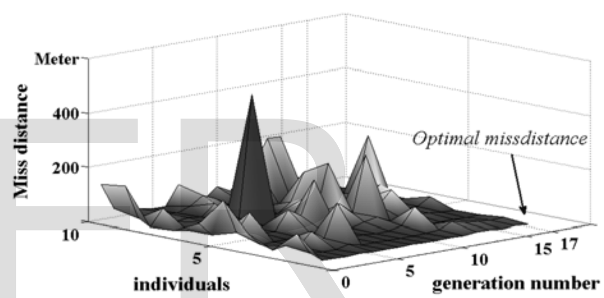

Fig.12- 3D plot of genetic algorithm search results

6.2. KF performance with differential geometry law

6.2.1 The effect of noise on kinematic boundaries

Fig.13 shows the influence of increasing noise factor on the performance of Kalman filter with differential geometry law. The figure shows that differential geometry law is very sensitive to increasing the gain of noise which causes a slight reduction in the effective area in the case of noise factor equal one, and great shrink in area at noise factor equal two.

6.2.2 Tuning of KF performance for noise factor=1

As mentioned before, tuning of Kalman filter by correction factors enhances the performance. In the case of differential geometry law there are four estimated values need to be tuned. The first value is the target acceleration and the rest values are target lead angle, closing velocity, and line of sight rate respectively.

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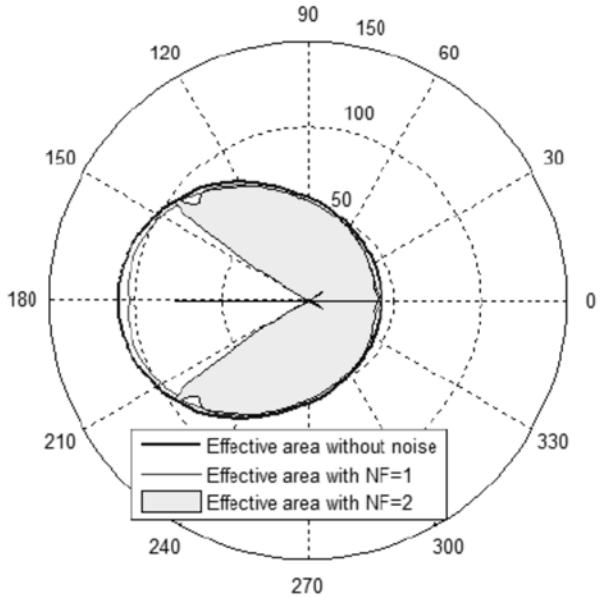

Fig.13 The effect of noise on effective areas

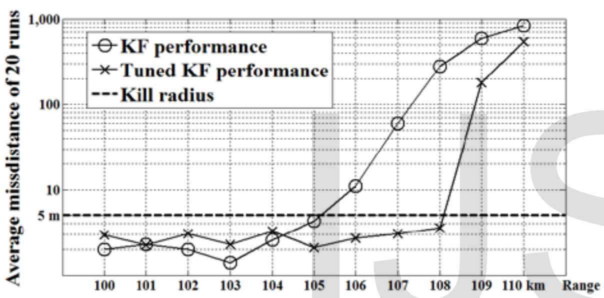

Fig.14 KF performance at long ranges (NF=1)

Fig.14 shows the results for both tuned and unturned values for Kalman filter for noise factor one and target heading 180 degrees. The results indicate increasing in the maximum effective range from 105 to 108 Km.

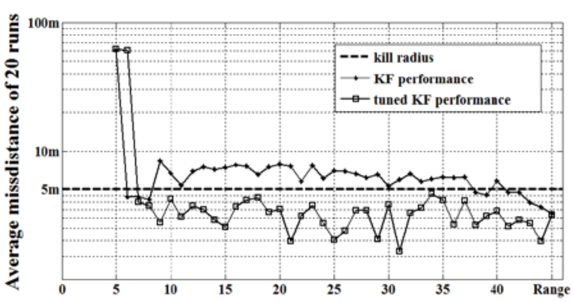

Fig.15 KF performance for low and medium ranges (NF=1)

Fig.15 shows the same results for low ranges which indicate improvement of minimum effective range from 41 Km to 7 Km by decreasing the miss distance smaller than 5 meters (the kill radius).

6.2.3 Tuning of KF performance for noise factor=2

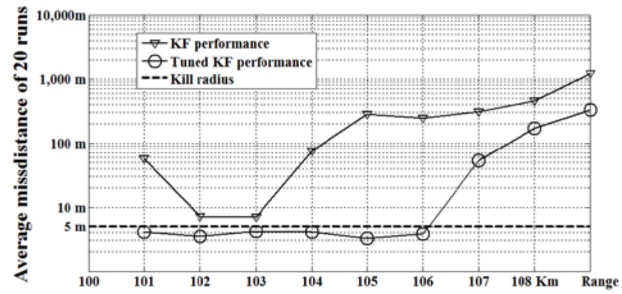

Fig.16 KF performance for long ranges (NF=2)

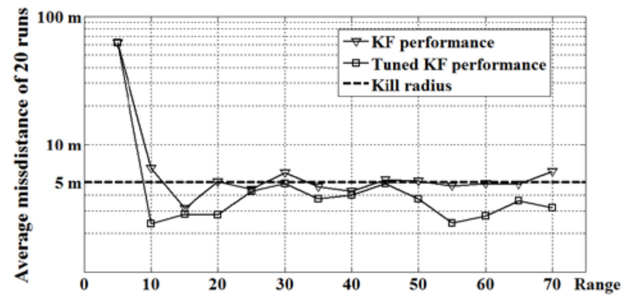

Fig.17 KF performance for low and medium ranges (NF=2)

Fig.16 and Fig.17 show the performance of Kalman filter and tuned Kalman filter when the noise factor =2 for long, medium, and low ranges. The target heading is 180 degree. The two figures show enhancement in performance due to using tuned Kalman filter.

TABLE I

DISTRIBUTION OF THE TUNING FACTORS ALONG THE RANGE

	Noise Factor=1		Noise Factor=2	
	First half	Second half	First half	Second half
Target acceleration	1.2	0.929	0.8516	0.84
Target lead angle	1.1	0.9806	1.084	1.2
Closing velocity	0.85	0.8	1.09	1.18
LOS rate	1.1355	0.81	1.161	0.968

In Table.1, the tuning factors for estimated values of target acceleration, target lead angle, closing velocity, and line of sight rate are shown for both noise factor one and two respectively and target heading=180˚.

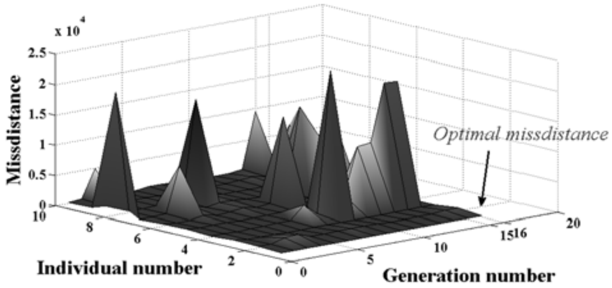

Fig.18.3D plot of genetic algorithm search results

Fig.18 shows 3D representation of genetic algorithm search results of target heading 180 degree, and noise factor=2. The optimum (minimum) average miss distance is the first individual of the sixteenth generation.

7 Conclusions

1-The performance of Kalman filter with proportional navigation law is enhanced by tuning factor which is found using genetic algorithm.
2- A genetic algorithm is used also to find the optimum tuning factor for differential geometry guidance law which reduces miss distance, increases the maximum effective range, and decreases the minimum effective range.
3- A differential geometry guidance law is very sensitive to increase the noise gain (noise factor) more than proportional navigation, thus tuning factors plays an important rule to enhance the performance especially with differential geometry law.

8 Summary

1- A noise causes undesirable effects on the performance of the guidance law.
2- A Kalman filter is used to get estimated values of the guidance law and reduces the bad effects of the noise.
3- The performance of Kalman filter is changed by multiply the estimated value by tuned factor lie between 0.8-1.2
4-A genetic algorithm is used to find the optimum values of tuned factors in every direction of target heading.
5- For future works, the tuning factors in all directions can be calculated and used as training data to train the neural networks such that these factors can be used online.

References

[1] C.K. Chui , G. Chen. " Kalman Filtering with Real-Time Applications". Fourth Edition, 2009
[2] C. Wren, A. Azarbayejani, T. Darrell, and A. Pentland. Pfinder. " Real-time tracking of the human body". IEEE Transactions on PAMI, 19(7):780–785, 1997.
[3] H. Ekenel and A. Pnevmatikakis. "Video-Based Face Recognition Evaluation " CHIL Project. International Conference of Pattern Recognition, 2006.
[4] D. Beymer, P. McLauchlan, B. Coifman, and J. Malik. "A real-time computer vision system for measuring traffic parameters". In Proc. CVPR, pages 495–501, 1997.
[5] G.D. Sullivan. "Model-based vision for traffic scene using the ground-plane constraint". Real-time Computer Vision,1994. D. Terzopoulos and C. Brown (Eds.).CUP
[6] D R Magee."Tracking Multiple Vehicles using Foreground, Background and Motion Models". December 2001 European Conference on Computer Vision, May 2002.
[7] Nilanjan Patra, Kaushik Halder, Avijit Routh, Abhro Mukherjee, Satyabrata Das, "Kalman Filter based Optimal Control Approach for Attitude Control of a Missile" International Conference on Computer Communication and Informatics, 2013.
[8] Osborn, Adam M." A study into the effects of Kalman filtered noise in advanced guidance laws of missile Navigation", Thesis, Naval Postgraduate School, 2014.
[9] Daniel Perh. "A study into advanced guidance laws using computational methods" Thesis, Naval Postgraduate School, 2011.
[10] L. Haupt Randy, Sue Ellen Haupt, " Practical Genetic Algorithms "(A John Wiley & Sons, INC.Publication, second edition, 2004)